

Unconventional Quantum Computing Devices

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Abstract: This paper investigates a variety of unconventional quantum computation devices, including fermionic quantum computers and computers that exploit nonlinear quantum mechanics. It is shown that unconventional quantum computing devices can in principle compute some quantities more rapidly than ‘conventional’ quantum computers.

Computers are physical: what they can and cannot do is determined by the laws of physics. When scientific progress augments or revises those laws, our picture of what computers can do changes. Currently, quantum mechanics is generally accepted as the fundamental dynamical theory of how physical systems behave. Quantum computers can in principle exploit quantum coherence to perform computational tasks that classical computers cannot [1-21]. If someday quantum mechanics should turn out to be incomplete or faulty, then our picture of what computers can do will change. In addition, the set of known quantum phenomena is constantly increasing: essentially any coherent quantum phenomenon involving nonlinear interactions between quantum degrees of freedom can in principle be exploited to perform quantum logic. This paper discusses how the revision of fundamental laws and the discovery of new quantum phenomena can lead to new technologies and algorithms for quantum computers.

Since new quantum effects are discovered seemingly every day, let’s first discuss two basic tests that a phenomenon must pass to be able to function as a basis for quantum computation. These are 1) The phenomenon must be nonlinear, and 2) It must be coherent. To support quantum logic, the phenomenon must involve some form of nonlinearity, e.g., a nonlinear interaction between quantum degrees of freedom. Without such a nonlinearity quantum devices, like linear classical devices, cannot perform even so simple a nonlinear operation as an AND gate. Quantum coherence is a prerequisite for performing tasks such as factoring using Shor’s algorithm [10], quantum simulation a la Feynman [11] and Lloyd [12], or Grover’s data-base search algorithm [13], all of which require extended manipulations of coherent quantum superpositions.

The requirements of nonlinearity and coherence are not only necessary for a phenomenon to support quantum computation, they are also in principle sufficient. As shown in [14-15], essentially any nonlinear interaction between quantum degrees of freedom suffices to construct universal quantum logic gates that can be assembled into a quantum computer. In addition, the work of Preskill *et al.* [18] on robust quantum computation shows that an error rate of no more than 10^{-4} per quantum logic operation allows one to perform arbitrarily long quantum computations in principle.

In practice, of course, few if any quantum phenomena are likely to prove sufficiently controllable to provide extended quantum computation. Promising devices under current experimental investigation include ion traps [5,7], high finesse cavities for manipulating light and atoms using quantum electrodynamics [6], and molecular systems that can be made to compute using nuclear magnetic resonance [8-9]. Such devices store quantum information on the states of quantum systems such as photons, atoms, or nuclei, and accomplish quantum logic by manipulating the interactions between the systems via the application of semiclassical potentials such as microwave or laser fields. We will call such devices ‘conventional’ quantum computers, if only because such devices have actually been constructed.

There is another sense in which such computers are conventional: although the devices described above have already been used to explore new regimes in physics and to create and investigate the properties of new and exotic quantum states of matter, they function according to well established and well understood laws of physics. Perhaps the most striking examples of the ‘conventionality’ of current quantum logic devices are NMR quantum microprocessors that are operated using techniques that have been refined for almost half a century. Ion-trap and quantum electrodynamic quantum computers, though certainly cutting edge devices, operate in a quantum electrodynamic regime where the fundamental physics has been understood for decades (that is not to say that new and unexpected physics does not arise frequently in this regime, rather that there is general agreement on how to model the dynamics of such devices).

Make no mistake about it: a conventional quantum logic device is the best kind of quantum logic device to have around. It is exactly *because* the physics of nuclear magnetic resonance and quantum electrodynamics are well understood that devices based on this physics can be used systematically to construct and manipulate the exotic quantum states that form the basis for quantum computation. With that recognition, let us turn to

‘unconventional’ quantum computers.

Perhaps the most obvious basis for an unconventional quantum computer is the use of particles with non-Boltzmann statistics in a regime where these statistics play a key role in the dynamics of the device. For example, Lloyd [16] has proposed the use of fermions as the fundamental carriers of quantum information, so that a site or state occupied by a fermion represents a 1 and an unoccupied site or state represents a 0. It is straightforward to design a universal quantum computer using a conditional hopping dynamics on an array of sites, in which a fermion hops from one site to another if only if other sites are occupied.

If the array is one-dimensional, then such a fermionic quantum computer is equivalent to a conventional quantum computer via the well-known technique of bosonization. If the array is two or more dimensional, however, a local operation involving fermions on the lattice cannot be mocked up by a local operation on a conventional quantum computer, which must explicitly keep track of the phases induced by Fermi statistics. As a result, such a fermionic computer can perform certain operations more rapidly than a conventional quantum computer. An obvious example of a problem that can be solved more rapidly on a fermionic quantum computer is the problem of simulating a lattice fermionic system in two or more dimensions. To get the antisymmetrization right in second quantized form, a conventional ‘Boltzmann’ quantum computer takes time proportional to $T\ell^{d-1}$ where T is the time over which the simulation is to take place, ℓ is the length of the lattice and d is the dimension, while a fermionic quantum computer takes time proportional to T . (Here we assume that the computations for both conventional and Fermionic quantum computers can take advantage of the intrinsic parallelizability of such simulations: if the computations are performed serially an additional factor of ℓ^d is required for both types of computer to update each site sequentially.)

As the lattice size ℓ and the dimension d grow large, the difference between the two types of computer also grows large. Indeed, the problem of simulating fermions hopping on a hypercube of dimension d as $d \rightarrow \infty$ is evidently exponentially harder on a conventional quantum computer than a Fermionic quantum computer. Since a variety of difficult problems such as the travelling-salesman problem and data-base search problem can be mapped to particles hopping on a hypercube, it is interesting to speculate whether fermionic computers might provide an exponential speed-up on problems of interest in addition to quantum simulation. No such problems are currently known, however. Fermionic computers could be realized in principle by manipulating the ways in which electrons and

holes hop from site to site on a semiconductor lattice (though problems of decoherence are likely to be relatively severe for such systems).

It might also be possible to construct bosonic computers using photons, phonons, or atoms in a Bose-Einstein condensate. Such systems can be highly coherent and support nonlinear interactions: phonons and photons can interact in a nonlinear fashion via their common nonlinear interaction with matter, and atoms in a Bose condensate can be made to interact via quantum electrodynamics (by introduction of a cavity) or by collisions. So far, however, the feature of Bose condensates that makes them so interesting from the point of view of physics — all particles in the same state — makes them less interesting from the point of view of quantum computation. Many particles in the same state, which can be manipulated coherently by a variety of techniques, explore the same volume of Hilbert space as a single particle in that state. As a result, it is unclear how such a bosonic system could provide a speed-up over conventional quantum computation. More promising than Bose condensates from the perspective of quantum computation and quantum communications, is the use of cavity quantum electrodynamics to ‘dial up’ or synthesize arbitrary states of the cavity field. Such a use of bosonic states is important for the field of quantum communications, which requires the ability to create and manipulate entangled states of the electromagnetic field.

A third unconventional design for a quantum computer relies on ‘exotic’ statistics that are neither fermionic nor bosonic. Kitaev has recently proposed a quantum computer architecture based on ‘anyons,’ particles that when exchanged acquire an arbitrary phase. Examples of anyons include two-dimensional topological defects in lattice systems of spins with various symmetries. Kitaev noted that such anyons could perform quantum logic via Aharonov-Bohm type interactions [19]. Preskill *et al.* have shown explicitly how anyonic systems could compute in principle [20], and Lloyd *et al.* have proposed methods of realizing anyons using superconducting circuits (they could also in principle be constructed using NMR quantum computers to mock up the anyonic dynamics in an effectively two-dimensional space of spins) [21]. The advantage of using anyons for quantum computation is that their nonlocal topological nature can make them intrinsically error-correcting and virtually immune to the effects of noise and interference.

As the technologies of the microscale become better developed, more and more potential designs for quantum computers, both conventional and unconventional, are likely to arise. Additional technologies that could prove useful for the construction of quantum

logic devices include photonic crystals, optical hole-burning techniques, electron spin resonance, quantum dots, superconducting circuits in the quantum regime, etc. Since every quantum degree of freedom can in principle participate in a computation one cannot *a priori* rule out the possibility of using currently hard to control degrees of freedom such as quark and gluon in complex nuclei to process information. Needless to say, most if not all of the designs inspired by these technologies are likely to fail. There is room for optimism that some such quantum computer designs will prove practicable, however.

The preceding unconventional designs for quantum computers were based on existing, experimentally confirmed physical phenomena (except in the case of non-abelian anyons). Let us now turn to designs based on speculative, hypothetical, and not yet verified phenomena. (One of the most interesting of these phenomena is large-scale quantum computation itself: can we create and systematically transform entangled states involving hundreds or thousands of quantum variables?) A particularly powerful hypothesis from the point of view of quantum computation is that of nonlinear quantum mechanics.

The conventional picture of quantum mechanics is that it is linear in the sense that the superposition principle is obeyed exactly. (Of course, quantum systems can still exhibit nonlinear interactions between degrees of freedom while continuing to obey the superposition principle.) Experiment confirms that the superposition principle is indeed obeyed to a high degree of accuracy. Nonetheless, a number of scientists including Weinberg have proposed nonlinear versions of quantum mechanics in which the superposition principle is violated. Many of these proposals exhibit pathologies such as violations of the second law of thermodynamics or the capacity for superluminal communication. Despite such theoretical difficulties, it is still possible that quantum mechanics does indeed possess a small nonlinearity, even if it currently seems unlikely. If a nonlinear operation such as that proposed by Weinberg can be incorporated in a quantum logic operation, then the consequences are striking: NP-complete problems can be solved easily in polynomial time [17]. Indeed, NP-oracle problems and all problems in $\#P$ can be solved in polynomial time on such a nonlinear quantum computer.

A general proof of this result is given in [17], however, a simple argument for why this is so can be seen as follows. Suppose that it is possible to perform a non-unitary operation on a single qubit that has a positive Lyapunov exponent over some region: i.e., somewhere on the unit sphere there exists a line of finite extent along which application of the operation causes nearby points to move apart exponentially at a rate $e^{\lambda\Delta\theta}$ proportional

to their original angular separation $\delta\theta$. Now consider a function $f(x)$ from N bits to one bit. We wish to determine whether or not there exists an x such that $f(x) = 1$, and if so, how many such x 's there are. Using the nonlinear operation with positive Lyapunov exponent, it is straightforward to construct a mapping leaves a point on the exponentially expanding line (call this point $|0\rangle$) fixed if there are no solutions to the equation $f(x) = 1$, and that maps the point to a nearby point $\cos(n/2^N)|0\rangle + \sin(n/2^N)|1\rangle$ along the line if there are exactly n solutions to the equation $f(x) = 1$. Repeated application of the nonlinear map can be used to drive the points apart at an exponential rate: eventually, at a time determined by the number of qubits N , the number of solutions n , and the rate of spreading λ , the two points will become macroscopically distinguishable, allowing one to determine whether or not there is a solution and if there is, how many solutions there are. The map f need only be applied once, and the amount of time it takes to reveal the number of solutions is proportional to N .

The fact that nonlinear quantum mechanics allows the straightforward solution of NP-complete and $\#P$ problems should probably be regarded as yet another strike against nonlinear quantum mechanics. Whether or not quantum mechanics is linear is a question to be resolved experimentally, however. In the unlikely event that quantum mechanics does turn out to be nonlinear, all our problems may be solved.

Finally, let us turn our attention to hypothetical quantum Theories of Everything, such as string theory. Such a theory must clearly support quantum computation since it supports cavity quantum electrodynamics and nuclear magnetic resonance. The obvious question to ask is then, does a Theory of Everything need to support anything *more* than quantum computation? So far as experimental evidence is concerned the answer to this question is apparently No: we have no evident reason to doubt that the universe is at bottom anything more than a giant, parallel, quantum information processing machine, and that the phenomena that we observe and attempt to characterize are simply outputs of this machine's ongoing computation. Of course, just how the universe is carrying out this computation is likely to remain a question of great interest for some time.

To summarize: Computers are physical systems, and what they can do in practice and in principle is circumscribed by the laws of physics. The laws of physics in turn permit a wide variety of quantum computational devices including some based on nonconventional statistics and exotic effects. Modifications made to the laws of physics have the consequence that what can be computed in practice and in principle changes. A particularly intriguing

variation on conventional physics is nonlinear quantum mechanics which, if true, would allow hard problems to be solved easily.

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